Fermi National Accelerator Laboratory

FN-307
0400

Further Parametric Studies of the Accelerator System for Heavy Ion Fusion

L.C. Teng

September 12, 1977

In earlier studies[1,2] we made the seemingly obvious assumption that the heaviest ion with the lowest charge state is most desirable. A closer examination showed that this is not entirely true. Furthermore, little attention was paid to the beam transport lines from the accelerator(s) to the reactor vessel. Simple calculations would reveal that the requirements of the bending dipoles in these transport lines and the final quadrupoles for focusing the beams onto the target are rather unrealistic for the examples[2] given. All these are re-examined in this paper.

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

We consider here only the case in which final longitudinal bunching of the beam (in addition to the bunching derived during acceleration) is necessary to obtain the required current. In this case a final bunching ring is needed. In principle the beam could be bunched in the final transport lines on the way to the reactor vessel, but in practice to obtain a significant degree of bunching would require either too long a beam transport or too large a momentum spread. This bunching ring will also serve as a beam distributing ring. The ring can be filled by the beam from a single accelerator. After bunching, the beam segments are extracted from symmetric points around the circumference and transported to strike simultaneously and symmetrically a target located at the center of the ring.

Even if final bunching is not needed, as long as the accelerated beam comes out of a single accelerator from a single spigot (e.g. a linac), the beam must be either split spatially or sectionalized temporally into branches. The branches must then be transported around to strike the target simultaneously from all directions. It is easy to see that the total length and bend angle of all the transport lines would very likely add up to be larger than those of the distributing ring. Hence even in this case a distributing ring may prove to be the most convenient and economical way to distribute and transport the final beams around.

1. Geometry of the bunching/distributing ring

If one is limited by a maximum available bending dipole field intensity the most convenient and economical geometry for the case of n (≥ 2) beams striking the target is as shown in Fig. 1.

The various geometrical parameters of these figures are

Total bend angle = $(n+1)\pi$

Total arc length = $(n+1)\pi\rho$

Total straight length = $2n\rho \sin\frac{\pi}{n}$

Circumference of ring = $2\pi\rho(1+\frac{n}{\pi} \sin\frac{\pi}{n})$

where ρ = radius of arc.

Here we considered only the planar cases and assumed that it is adequate for the beams to strike the target symmetrically in a plane.

2. Targeting requirement

The targeting requirements are specified by the following 4 parameters:

W = Total energy on target

P = Power on target = Rate of energy deposited on target

E = Specific energy on target = Energy deposited per gram of target

r = Beam spot radius on target

In this paper we will consider two sets of these parameters.

Ion Fusion Power Plant (IFPP)[1]

$$\begin{cases} W = 10 \text{ MJ} \\ P = 600 \text{ TW} \\ E = 30 \text{ MJ/g} \\ r \geq 0.1 \text{ cm} \end{cases}$$

HIDE (Target Coupling Experimental System)

$$\begin{cases} W = 100 \text{ kJ} \\ P = 50 \text{ TW} \\ E = 300 \text{ kJ/g} \\ r \geq 0.1 \text{ cm} \end{cases}$$

3. Space charge effect

In the bunching/distributing ring the tune shift caused by space charge forces is given by[2]

$$\Delta\nu = \frac{(qe)^2}{mc^2} \frac{I}{qec} \frac{R}{\varepsilon_t/\pi} \frac{1}{(\beta\gamma)^2} \tag{1}$$

where

qe = charge of ion $\quad \beta = \frac{1}{c}$ (speed)

m = mass of ion $\quad \gamma = \frac{1}{mc^2}$ (total energy)

I = electric current of beam

ε_t/π = normalized transverse emittance of beam

$R = \frac{1}{2\pi}$ (circumference of ring).

First we transform $\Delta\nu$ into a form which is explitictly independent of the ion specie. We have

$$\frac{I}{qe} = \frac{P/n}{T} = \frac{P/n}{mc^2(\gamma-1)} = \text{particle current} \tag{2}$$

where n = number of beams on target, T = kinetic energy of ion, and

$$R = \frac{pc}{qeB_D} = \frac{mc^2}{qeB_D}\,\beta\gamma = \text{ring radius} \tag{3}$$

where p = momentum of ion, B_D = average bending dipole field in ring. In earlier studies we considered ε_t/π as given by the focusing requirement on target. This led to rather unreasonable demands on the final focusing quadrupoles. It is, therefore, more appropriate to consider ε_t/π as given by realistic final quadrupoles. For this, we have

$$\varepsilon_t/\pi = \beta\gamma r\theta \cong \beta\gamma r \frac{qeB_Q\ell_Q}{pc} = \frac{qeB_Q\ell_Q}{mc^2}\, r \tag{4}$$

where θ = half convergent angle on target, $B_Q\ell_Q$ = pole field times length of final quadrupole. Here we considered only the focusing plane of a single quadrupole. In reality we must use at least a quadrupole doublet for focusing in both planes. This must be kept in mind when one assigns reasonable values for $B_Q \ell_Q$. Substituting Eqs. (2), (3) and (4) in Eq. (1) we get

$$\Delta\nu = \frac{P/n}{B_D (B_Q \ell_Q) c} \frac{1}{\beta\gamma(\gamma-1)} \frac{1}{r} . \tag{5}$$

One set of consistent units for this equation is

P in erg/sec

B in Gauss

all lengths in cm.

The beam spot radius r on target is given by the requisit specific energy deposition E and the range λ of the ions in target through

$$E = \frac{W}{\pi r^2 \lambda} \quad \text{or} \quad \frac{1}{r} = \sqrt{\frac{\pi E}{W}} \sqrt{\lambda} . \tag{6}$$

Implied in this equation is the assumption that whatever the number of beams the total energy W carried by all the beams can be deposited in the target volume $\pi r^2 \lambda$. Substituting Eq. (6) in Eq. (5) we get, finally

$$\Delta\nu = \frac{P\sqrt{\pi E/W}}{B_D (B_Q \ell_Q) c} \frac{F}{n} \qquad F \equiv \frac{\sqrt{\lambda}}{\beta\gamma(\gamma-1)} . \tag{7}$$

In this form the explicit dependence on the charge ge and mass m of the ions vanishes and the dependence on the ion specie is only implicit through the factor

$$F \equiv \frac{\sqrt{\lambda}}{\beta\lambda(\lambda-1)} .$$

This factor is plotted against the normalized kinetic energy γ-1 in Fig. 2 for Ca^{40}, I^{127} and U^{238} using the range information provided by R. Bangerter.[3] Several interesting features of Eq. (7) are worth mentioning.

a. For the same kinetic energy $T = mc^2(\gamma-1)$ the factor F, hence $\Delta\nu$, is smaller for a lighter ion or, conversely, for the same $\Delta\nu$, hence the same F, the required energy T is lower for a lighter ion. This is because for lighter ions the larger relativistically normalized energy parameters in the denominator override the increase in range in the numerator. Hence as far as $\Delta\nu$ is concerned within the ranges of ion species and energy covered in Fig. 2 lighter ions are preferred.

b. One must also consider the ring aperture, namely the emittance of the beam. To investigate this we get from Eqs. (6) and (7)

$$\frac{1}{r} = \sqrt{\frac{\pi E}{W}}\ \beta\gamma(\gamma-1)F \tag{8}$$

which when substituted in Eq. (4) gives for the un-normalized emittance

$$\frac{\varepsilon_t/\pi}{\beta\gamma} = \frac{qe}{mc^2\beta\gamma} B_Q \ell_Q r = \frac{B_Q \ell_Q}{RB_D} r$$

$$= \frac{B_Q \ell_Q}{RB_D} \frac{1}{F\sqrt{\pi E/W}} \frac{1}{\sqrt{(\gamma+1)(\gamma-1)^3}} \tag{9}$$

where we have used Eq. (3). Hence for the same F and the same available field $R\frac{\varepsilon_t/\pi}{\beta\gamma}$ reduces as one goes to a lighter ion. By adjusting the charge qe of the ion we can apply this reduction to either the ring radius or the emittance or both. Therefore, as far as emittance is concerned lighter ions are also preferred.

c. The tune shift $\Delta\nu$ as given by Eq. (7) is independent of the charge qe of the ions. This is because at higher charge the higher space charge forces in the numerator are compensated by the higher magnetic confinement forces in the denominator. The choice of the charge state q of the ion depends, therefore, only on considerations of the ring radius and the beam emittances as given by Eq. (9).

4. Design procedure and considerations

The design considerations and procedure can now be summarized as follows:

a. The desired P and E/W are given by the targeting requirements.

b. In choosing B_D one should keep in mind the ring geometries shown in Fig. 1. This gives

$$B_D = \frac{1}{1+\frac{n}{\pi}\sin\frac{\pi}{n}} \quad \text{(average field in arc)}$$

$$\cong \frac{1}{1+\frac{n}{\pi}\sin\frac{\pi}{n}} \quad \text{(~80\% of field in dipole)}.$$

Therefore B_D should generally be no greater than half of the field in the dipoles, namely ~10 kG for conventional magnets or ~20-25 kG for superconducting magnets.

c. In choosing $B_Q \ell_Q$ one should keep in mind that we need at least a doublet for focusing in both transverse planes. Hence ℓ_Q should be no larger than 1-2 m. The pole field B_Q could be ~14 kG for conventional magnets or ~40-50 kG for superconducting magnets.

d. If the beam has to be stored in the ring for hundreds or more revolutions the maximum allowable $\Delta\nu$ is $\frac{1}{4}$ in order to avoid major resonances. On the other hand, A. Maschke[4] has shown that if the final bunching is sufficiently fast (tens of revolutions) even integer resonances can be crossed without noticeable deterioration of the transverse emittance. In this case $(\Delta\nu)_{max}$ of several units may even be allowable. The bunching/distributing ring could also be used as a synchrotron for accelerating the ions to the final energy. Throughout acceleration, then, $\Delta\nu$ of the rf-bunched beam must be $<\frac{1}{4}$. Only during the additional fast bunching on the flat-top can this upper limitation of $\frac{1}{4}$ for $\Delta\nu$ be removed.

e. Having chosen B_D, $B_Q \ell_Q$ and $(\Delta\nu)_{max}$; and with the given P and E/W we get from Eq. (7) the maximum allowed value F_{max}/n. In addition, for a given value of F the lower limit (0.1 cm) of r gives an upper limit for γ-1 through Eq. (8). Within these two bounds one should choose the lightest ion from the curves of Fig. 2. The optimum n is that for which these two bounds would permit the use of the lightest ion overall. Of course, the increased cost associated with a larger number of beam transports is also an important consideration. In addition the charge-exchange collision cross-sections for specific ion types should also be considered if the ion lifetime proves to be a concern.

f. The charge number q should be large to reduce the ring radius R. At the same time the beam emittance and hence the ring aperture as given by Eq. (9) must be reasonable. Again, if relevant, the charge-exchange collision cross section should also be considered in choosing the charge state.

This procedure will give a rough cut of the gross parameters for the case in which a final beam bunching/distributing ring is used. The parameters so obtained should be reasonably optimal.

5. Examples

We now demonstrate this design procedure using the two systems with targeting parameters given in section 2 above.

a. IFPP as described in FN-302 (reference 2)

First we check that starting with the assumptions made in FN-302 this procedure will indeed lead to the design developed there. In that paper we ignored the final transport to the reactor vessel, and chose

$$B_D = 65\% \times 50 \text{ kG} \cong 30 \text{ kG} = 3\text{x}10^4 \text{ G}$$

and

$$\varepsilon_t/\pi \cong 4 \text{ cm-mrad.}$$

For r = 0.1 cm and U_{238}^{+1} ions (mc^2 = 222 GeV, q = 1) this emittance gives through Eq. (4)

$$B_Q \ell_Q \cong 300 \text{ kG m} = 3\text{x}10^7 \text{ G cm.}$$

[Even for B_Q = 50 kG this requires ℓ_Q = 6 m. Considering that quadrupole doublets are needed for focusing in both planes this value of $B_Q \ell_Q$ is definitely too large.]

In FN-302 we also assumed that only half of the total beam energy W is deposited in a volume $\pi r^2 \lambda$ of the target. Therefore

$$\frac{E}{W} = \frac{30 \text{ MJ/g}}{5 \text{ MJ}} = 6 \text{ g}^{-1}.$$

Other parameters assumed are

$$P = 600 \text{ TW} = 6\text{x}10^{21} \text{ erg/sec}$$

$$n = 10$$

$$(\Delta\nu)_{max} = \frac{1}{4} .$$

Eq. (7) gives

$$F_{max} = \frac{\sqrt{\lambda}}{\beta\gamma(\gamma-1)} = \frac{10\text{x}3\text{x}10^4\text{x}3\text{x}10^7\text{x}3\text{x}10^{10}}{4\text{x}6\text{x}10^{21}\text{x}\sqrt{6\pi}} \text{ g}^{\frac{1}{2}}/\text{cm} = 2.6 \text{ g}^{\frac{1}{2}}/\text{cm}.$$

The U curve of Fig. 2 then gives for the required energy

$$\gamma-1 \cong 0.65 \quad \text{or} \quad T \cong 150 \text{ GeV}$$

agreeing with that given in FN-302.

b. IFPP from this procedure

When the final beam transports to the reactor vessel are taken into account reasonable choices of field intensities are

$$B_D = 20 \text{ kG} = 2\text{x}10^4 \text{ G}$$

$$B_Q \ell_Q = 50 \text{ kG x } 1 \text{ m} = 5\text{x}10^6 \text{ G cm.}$$

Assuming the final fast bunching on the flat-top to give a factor 4 we can take

$$(\Delta\nu)_{max} = 1.$$

We further assume that the total beam energy of W = 10 MJ is deposited in a target volume of $\pi r^2 \lambda$ and obtain

$$\frac{E}{W} = \frac{30 \text{ MJ/g}}{10 \text{ MJ}} = 3 \text{ g}^{-1}.$$

Eq. (7) then gives

$$\frac{F_{max}}{n} = \frac{2\text{x}10^4\text{x}5\text{x}10^6\text{x}3\text{x}10^{10}}{6\text{x}10^{21}\text{x}\sqrt{3\pi}} \text{ g}^{\frac{1}{2}}/\text{cm} = 0.16 \text{ g}^{\frac{1}{2}}/\text{cm}.$$

To get the same $F_{max} = 2.6 \text{ g}^{\frac{1}{2}}/\text{cm}$ we need 16 beams and for $r>0.1$ cm Eq. (8) gives $\gamma-1<0.822$. For this illustrative example we will not bother to optimize n.

It is a little difficult to extrapolate and interpolate the curves in Fig. 2 accurately, but it seems safe to take

$$mc^2 = 175 \text{ GeV} \qquad (\text{Os}^{188}!)$$

and $\gamma-1 = 0.8$

$T = 140$ GeV $\qquad (\beta\gamma = 1.497)$.

Eq. (8) then gives

$$r = 0.105 \text{ cm}$$

and Eq. (9) gives

$$R\frac{\varepsilon_t/\pi}{\beta\gamma} = 26.2 \text{ cm}^2.$$

With a charge number $q = 3$ we get

$$R = 146 \text{ m} \quad \text{and} \quad \frac{\varepsilon_t/\pi}{\beta\gamma} = 1.80 \text{ cm-mrad}.$$

For this ring we can use 24 m long FODO cells with $\beta_{max} \cong 40$ m. Thus, the beam radius in only ~2.7 cm which is quite modest. On the other hand, if the ring is also used as a synchrotron its aperture must be determined by the larger beam size at injection.

The total particle current is 4286 A or 268 A in each of the 16 beams. With charge number 3 the electric current is 804 A. With

$$q = 3 \qquad \frac{e^2}{mc^2} = 0.823\text{x}10^{-18} \text{ cm}$$

$$I = 804 \text{ A} \qquad R = 1.46\text{x}10^4 \text{ cm}$$

$$\varepsilon_t/\pi = 2.7\text{x}10^{-3} \text{ cm} \qquad \beta\gamma = 1.497$$

Eq. (1) gives indeed $\Delta\nu = 1.0$ as originally assumed.

With an emittance of $\frac{\varepsilon_t/\pi}{\beta\gamma} = 1.80$ cm-mrad and a beam spot radius on target of $r = 0.105$ cm if the final quadrupole is 10 m away from the target the quadrupole aperture radius r_Q will be

$$r_Q = \frac{10 \text{ m x } 1.80 \text{ cm-mrad}}{0.105 \text{ cm}} = 17 \text{ cm}$$

a quite reasonable value.

c. HIDE

We will try with conventional magnets and take

$$B_D = 10 \text{ kG} = 1\text{x}10^4 \text{ G}$$

$$B_Q \ell_Q = 14 \text{ kG x } 1.4 \text{ m} = 2\text{x}10^6 \text{ G cm.}$$

Again, we assume that the bunching/distributing ring is also used as a synchrotron. Taking the final fast bunching factor to be 6 we can allow the final tune shift to be as large as

$$(\Delta\nu)_{max} = 1.5.$$

Eq. (7) then gives

$$\frac{F_{max}}{n} = \frac{1.5\text{x}10^4\text{x}2\text{x}10^6\text{x}3\text{x}10^{10}}{5\text{x}10^{20}\text{x}\sqrt{3\pi}} \; g^{\frac{1}{2}}/\text{cm} = 0.59 \; g^{\frac{1}{2}}/\text{cm}.$$

Again without bothering to optimize n we shall choose n = 8 which gives $F_{max} = 4.7 \; g^{\frac{1}{2}}/\text{cm}$. Eq. (8) then gives for $r>0.1$ cm the upper limit $\gamma-1<0.572$.

Extrapolating and interpolating between the curves of Fig. 2 indicate that we can safely choose an ion with

$$mc^2 = 100 \text{ GeV} \qquad (Ag^{107}!)$$

and

$$\gamma-1 = 0.55$$

$$T = 55 \text{ GeV} \qquad (\beta\gamma = 1.184).$$

Eq. (8) then gives

$$r = 0.107 \text{ cm}$$

and Eq. (9) gives

$$R\frac{\varepsilon_t/\pi}{\beta\gamma} = 21.3 \text{ cm}^2.$$

With a charge number q = 5 we get

$$R = 79 \text{ m} \quad \text{and} \quad \frac{\varepsilon_t/\pi}{\beta\gamma} = 2.7 \text{ cm-mrad.}$$

For such a ring one can use 16 m long FODO cells giving $\beta_{max} \cong 27$ m. Thus the beam radius is only ~2.7 cm which is quite modest. Of course, since the ring is also used as a synchrotron the aperture must be sized to the larger beam radius at injection.

The total particle current is 909 A or 113.6 A in each of 8 beams. With charge number 5 the electric current is 113.6x5 A = 568 A. Substituting

$$q = 5 \qquad \frac{e^2}{mc^2} = 1.44\times10^{-18} \text{ cm}$$

$$I = 568 \text{ A} \qquad R = 7900 \text{ cm}$$

$$\varepsilon_t/\pi = 3.2\times10^{-3} \text{ cm} \qquad \beta\gamma = 1.184$$

in Eq. (1) we get indeed $\Delta\nu = 1.5$ agreeing with the starting design assumption.

With an emittance of $\frac{\varepsilon_t/\pi}{\beta\gamma} = 2.7$ cm-mrad and a beam spot radius of r = 0.107 cm if the final quadrupole is 10 m away from the target the quadrupole aperture radius r_Q will be

$$r_Q = \frac{10 \text{ m x } 2.7 \text{ cm-mrad}}{0.107 \text{ cm}} = 25 \text{ cm}$$

which is large but not unreasonable.

References

1. Final Report of ERDA Summer Study of Heavy Ions for Inertial Fusion, LBL-5543, Dec. 1976

2. L.C. Teng, "A 'Conventional' Accelerator System for Inertial Fusion Using Heavy Ion Beams", Fermilab Report FN-302, Jan. 1977

3. R.O. Bangerter, "Target Requirement for Storage Ring Inertial Confinement Fusion", unpublished report distributed at the ERDA Summer Study of Heavy Ions for Inertial Fusion (June 1976) (also contained in reference 1).

4. G. Danby, E. Gill, J. Keane and A.W. Maschke, "Preliminary Results of 200 MeV Bunching Experiments", Brookhaven National Laboratory Report BNL 50643, March 1971

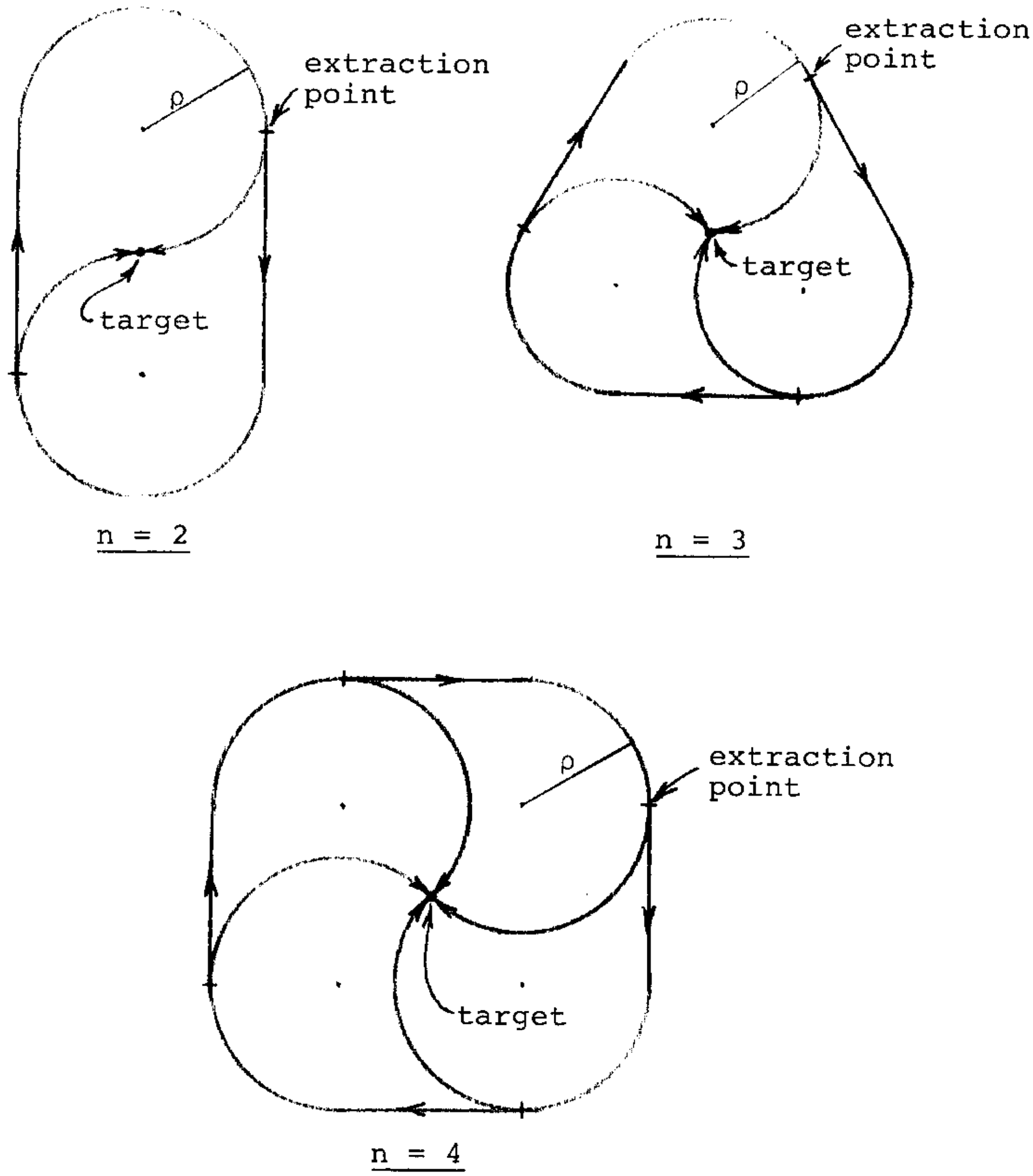

Figure 1 Geometry of bunching/distributing ring and beam transport lines to reactor vessel (n = number of beams on target)

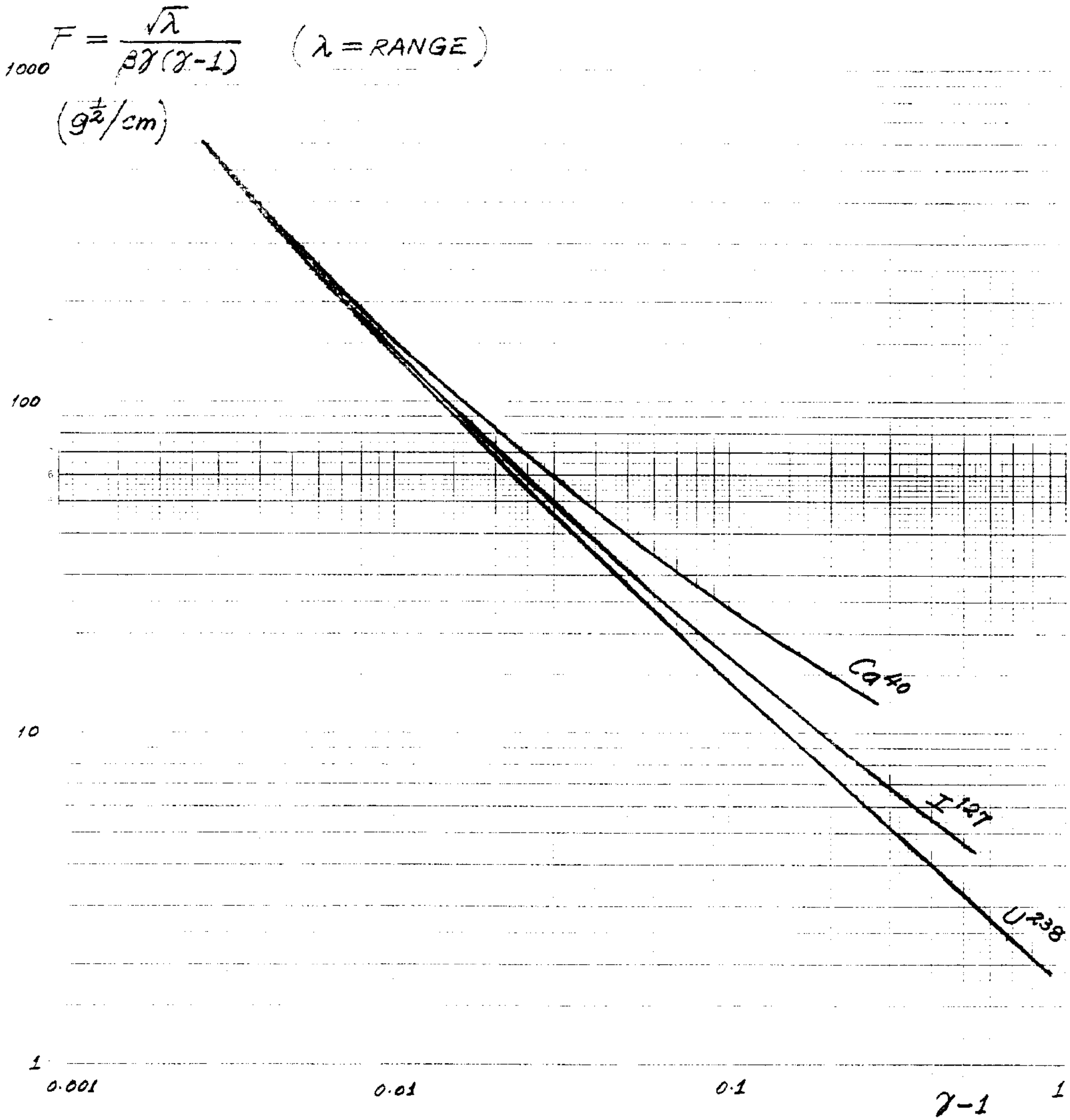

Figure 2 Range-energy relations for U^{238}, I^{127} and Ca^{40} ions in gold - F versus γ-1.